

Problem: Find the coefficients \underline{a} , \underline{b} , and \underline{c} so that the graph of $f(x) = ax^2 + bx + c$ passes through the points $(1, 2)$, $(-1, 6)$, and $(2, 3)$. From Linear Algebra by Jim Hefferon

Solution: $f(1) = 2$, $f(-1) = 6$, and $f(2) = 3$

So we set up a linear system.

$$f(1) = a(1)^2 + b(1) + c = 2$$

$$f(-1) = a(-1)^2 + b(-1) + c = 6$$

$$f(2) = a(2)^2 + b(2) + c = 3$$

So to solve this system, we set up an augmented matrix and solve it, using Gauss-Jordan elimination.

$$\begin{array}{ccc|c|} 1 & 1 & 1 & 2 & | \\ 1 & -1 & 1 & 6 & | \\ 4 & 2 & 1 & 3 & | \end{array}$$

$$\begin{aligned} R2 + -1(R1) = & \begin{array}{cccc|ccc} 1 & 1 & 1 & 2 & & & \\ 0 & -2 & 0 & 4 & & & \\ 4 & 2 & 1 & 3 & & & \end{array} \end{aligned}$$

$$\begin{aligned} R3 + -4(R1) = & \begin{array}{cccc|ccc} 1 & 1 & 1 & 2 & & & \\ 0 & -2 & 0 & 4 & & & \\ 0 & -2 & -3 & -5 & & & \end{array} \end{aligned}$$

$$\begin{aligned} R3 + -1(R2) = & \begin{array}{cccc|ccc} 1 & 1 & 1 & 2 & & & \\ 0 & -2 & 0 & 4 & & & \\ 0 & 0 & -3 & -9 & & & \end{array} \end{aligned}$$

$$\begin{aligned} R1 + 1/2(R2) = & \begin{array}{cccc|ccc} 1 & 0 & 1 & 4 & & & \\ 0 & -2 & 0 & 4 & & & \\ 0 & 0 & -3 & -9 & & & \end{array} \end{aligned}$$

$$\begin{aligned} R1 + \frac{1}{3}(R3) = & \begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -2 & 0 & 4 \\ 0 & 0 & -3 & -9 \end{array} \end{aligned}$$

$$\begin{aligned} R2 * -\frac{1}{2}(R2) = & \begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -3 & -9 \end{array} \end{aligned}$$

$$\begin{aligned} R3 * -\frac{1}{3}(R3) = & \begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array} \end{aligned}$$

So, then $a = 1$, $b = -2$, and $c = 3$. We have found the coefficients of the quadratic equation that passes through the three points, $(1, 2)$, $(-1, 6)$, and $(2, 3)$.

$$f(x) = 1x^2 + -2x + 3, \text{ or simply } f(x) = x^2 - 2x + 3$$